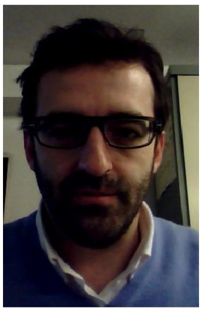
BOND MODELLING OF PRESTRESSED CONCRETE DURING THE PRESTRESSING FORCE RELEASE

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Abstract

This paper presents an analytical model for simulating the bond between steel and concrete, in precast prestressed concrete elements, during the prestressing force release. The model establishes a relationship between bond stress, steel and concrete stress and slip in such concrete structures. This relationship allows us to evaluate the bond stress in the transmission zone, where bond stress is not constant, along the whole prestressing force release process. The model is validated with the results of a series of tests and is extended to evaluate the transmission length. This capability has been checked by comparing the transmission length predicted by the model and one measured experimentally in a series of tests.

Keywords: Bond, prestressed concrete, transmission length, modelling, bond strength

1 Introduction

Precast prestressed concrete elements are widely used for construction in Europe. A frequent problem of the precast industry is to evaluate the real transmission length in precast prestressed concrete structural elements. The semi-empirical formulae proposed by codes are usually thought for conventional concrete and usual cast conditions, but high performance concrete (high-strength, self- compacting, etc.) and non-usual cast conditions (*v.gr.* accelerated curing processes) are becoming more and more frequent. In these cases experimental measurement is needed, though the standardised methods are expensive and so difficult to apply by industry. Analytical and numerical models, based on parameters measured experimentally with tests being simpler than complete transmission length tests, would be welcomed. This paper presents an analytical model for steel-concrete bond when the prestressing force is transmitted by releasing the steel (wire or strand). The model is applied to evaluate the transmission length.

2 Theoretical background

The prestressing process of the precast prestressed concrete includes: a) steel wire prestressing, and b) prestressing force transmission after an accelerated curing process of concrete. Let be a prism of concrete with a single prestressed wire placed in the prism longitudinal axis (**Fig. 1**). Be P_0 the initial prestressing force in the wire and, for a given instant, the force at the end of the wire P_0-ΔP, where ΔP is the released force. **Fig. 2** shows the balance of stresses in slice and **Fig. 3** in a part of the specimen. The equilibrium forces (**Fig. 2**) leads to the following differential equation:

$$\frac{\partial \Delta \sigma_x}{\partial x} = -\frac{p_e}{A_s}\tau \tag{1}$$

where $\Delta\sigma_x$ is the wire stress variation between sections x and $x+dx$, p_e is the wire perimeter, A_s is the area of wire section and τ is the mean tangential stress between wire and concrete in the slice.

The slip between wire and concrete at any section may be expressed (**Fig. 3**), as follows:

$$\frac{ds}{dx} = \frac{\Delta\sigma_x}{E_s} - \frac{\sigma_c}{E_c} \tag{2}$$

where s is the slip between wire and concrete, dx is the slice thickness, E_s and E_c are Young's modulus of the steel and concrete, respectively, and σ_c is the concrete normal stress at x section.

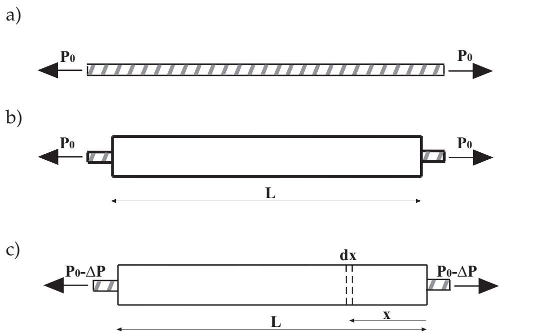

Fig.1. Manufacturing process of the prestressed concrete prism: a) alone prestressed wire, b) wire with cast concrete before prestressing force release, c) prestressing force release.

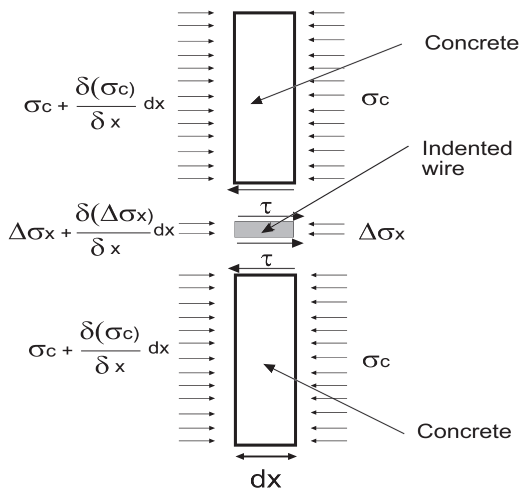

Fig.2. Balance of stresses in a slice of *dx* thickness.

To take into account the concrete confinement of the wire, the analogy of a thick-walled cylinder is adopted. **Fig. 4** shows the sketch of the model. R_1 is the inner radius and R_2 is the outer radius. Then, the circumferential strain of any point may be expressed as follows:

$$\text{for steel: } \varepsilon_{\theta s} = \frac{\sigma_{\theta s} - \nu_s(\sigma_{rs} + \sigma_{zs})}{E_s} \quad \text{, for concrete: } \varepsilon_{\theta c} = \frac{\sigma_{\theta c} - \nu_c(\sigma_{rc} + \sigma_{zc})}{E_c} \tag{3}$$

The σ_{zs} stress is caused by the released force ΔP, and according to **Fig. 4**, the stress in the steel may be identified as:

$$\sigma_{rs} = -\sigma_b; \quad \sigma_{\theta s} = -\sigma_b; \quad \sigma_{zs} = -\Delta\sigma_x \tag{4}$$

and in concrete:

$$\sigma_{rc} = -\sigma_b; \quad \sigma_{\theta c} = H\,\sigma_b; \quad \sigma_{zc} = \sigma_c$$

$$\text{with} \quad H = \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} \tag{5}$$

where σ_b is the normal stress between steel and concrete, and it is related to tangential stress τ, by means of the Tepfers`s equation [1]:

$$\tau = \sigma_b \cot\alpha \tag{6}$$

Combining equations (3), (4) and (5) the following is obtained:

$$\sigma_b = J\ \Delta\sigma_x - M\ \Delta\sigma_0 \qquad \tau = B_1\ \Delta\sigma_x - B_2\ \Delta\sigma_0 \qquad \frac{\partial \Delta\sigma_x}{\partial x} = -B\ \Delta\sigma_x + C$$

$$\text{with} \begin{cases} J = \dfrac{\nu_s E_c + \dfrac{A_s}{A_c}\nu_c E_s}{(1-\nu_s)E_c + (H+\nu_c)E_s} \\ M = \dfrac{\nu_c E_s \dfrac{A_s}{A_c}}{(1-\nu_s)E_c + (H+\nu_c)E_s} \end{cases} \qquad \text{with} \begin{cases} B_1 = \dfrac{J}{\tan\alpha} \\ B_2 = \dfrac{M}{\tan\alpha} \end{cases} \qquad \text{with} \begin{cases} B = \dfrac{p_e}{A_s} B_1 \\ C = \dfrac{p_e}{A_s} B_2\ \Delta\sigma_0 \end{cases} \tag{7, 8 \& 9}$$

whose analytical solution is:

$$\Delta\sigma_x = \frac{C}{B} + k\ e^{-Bx} \tag{10}$$

where k is an integration constant to be determined for each particular case. Detailed information about the model can be found in [2].

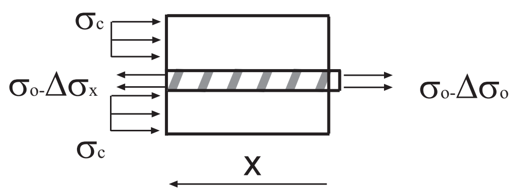

Fig. 3. Balance of forces in a part of the specimen with x length, measured from the end.

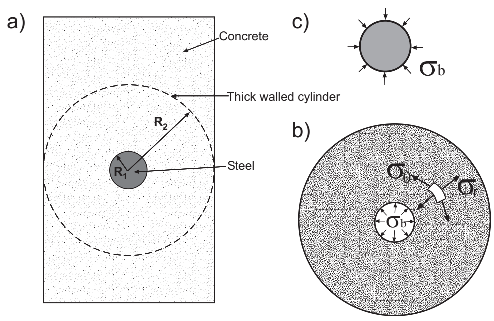

Fig. 4. Interaction between concrete and steel at the interface: a) thick-walled cylinder approach of the prism cross-section, b) stresses in concrete, c) stresses in steel due to confinement.

3 Model validation

The proposed analytical model is validated with the experimental results of [3]. **Fig. 5** shows the geometry and the dimensions of the specimens. The tests were performed with indented wires Y 1770 C with 4mm (E = 226 GPa, $\sigma_{0,2\%}$ = 1,755 MPa and σ_u = 1,935 MPa). **Fig 6**. compares the experimental curves of slip at the end of the specimen versus the released load with the model prediction.

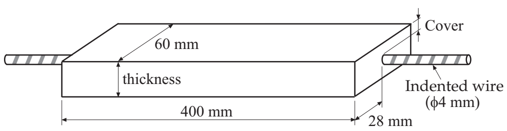

Fig. 5. Geometry and dimensions of the specimens from Gálvez et al [3

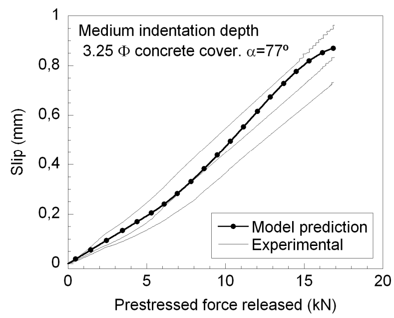

Fig. 6. Experimental results [3] and numerical prediction of the released prestressing force *versus* slip.

4 Transmission length calculation

To validate the model for determining the transmission length, the results of Russell and Burns [4], who manufactured pretensioned prisms of 102mm x 127mm x 3,660mm with seven-wire strands, were analytically simulated. They used single strands ϕ 12.7mm and 15.2mm, placed in the barycentre of the cross-section. The material properties were: E = 27 GPa, f_{ci} = 28 MPa and ν = 0.2, for concrete; and E = 194 GPa and ν = 0.3, for steel. Young's Modulus of concrete was estimated according to Model Code specifications. **Fig. 7** shows the experimental results and the analytical prediction.

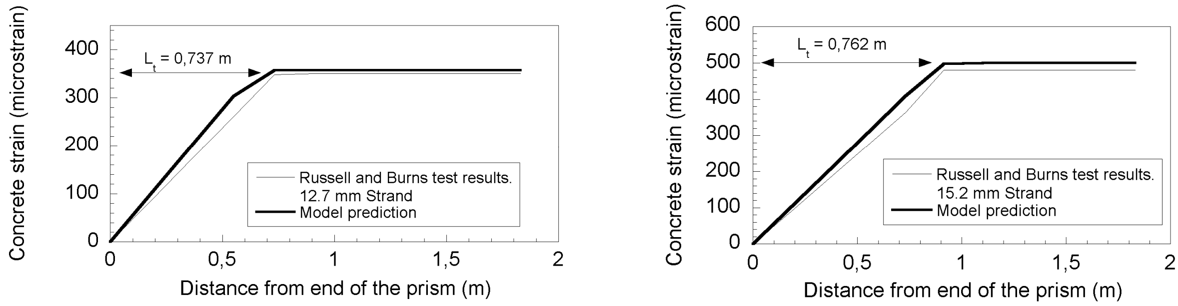

Fig. 7. Strain profile and transmission length. Comparison of model with experimental results from Russell and Burns [4] (12.7mm and 15.2 mm seven-wire strand specimens).

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